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TITLE: INAPPROPRIATENESS OF THE FINITE LARMOR RADIUS MODEL FOR THE TILTING MODE IN FIELD REVERSED CONFIGURATIONS

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<u>Inappropriateness of the Finite Larmor Radius Model for the Tilting Mode in Field Reversed Configurations</u>, J. L. Schwarzmeier, <u>LANL</u>, and C. E. Seyler, <u>Cornell Univ.</u> Ithaca, New York 14853

## I. INTRODUCTION

In an earlier publication[1] we provided substantial evidence that according to the predictions of magnetohydrodynamics (MHD), any realistic field-reversed configuration (FRC) equilibrium should be very unstable to internal tilting. For the parameters of the Los Alamos FRX-B experiment the e-folding time of the instability is about 1 $\mu s$[1]. However, FRX-B shows no signs of tilting during the 20 $\mu s$ quiescent phase of the experiments that precedes the onset of the n=2 rotational instability. Thus evidently there is a major disagreement between the experiments and the predictions of MHD regarding the tilting instability. In this work we present numerical results of the finite Larmor radius (FLR) treatment of the tilting mode as described by Seyler and Barnes[2]. We have two principal results. First, the FLR theory of tilting has singularities at the magnetic axis that make application of the theory unphysical. Second, numerical results will be presented showing that for current experiments the magnetic moments of the ions are poorly conserved at the tips of the flux surfaces in an FRC, and therefore there is <u>a priori</u> reason to view an FLR theory as suspect for global modes in FRCs.

## II. MHD RESULTS FOR TILTING

Reference 1 contains a stability study of realistic FRC equilibria. The conclusion of that work is that MHD profile effects cannot account for the observed stability of the experiments, since all equilibria investigated were roughly equally unstable to tilting. We assume that three properties of the MHD eigenfunction hold approximately in our FLR calculation: that the displacement in the r-z plane is axial (highly elongated FRC); that the axial displacement is rigid, $\xi_z = \xi_z(\psi)$ (elliptical-like equilibria); and, that $\xi_z(\psi=0)=0$ , where $\psi=0$ is the separatrix (internal mode).

## III. FLR THEORY OF TILTING

In this section we present the dispersion functional appropriate to the Vlasov-fluid model as derived by Seyler and Barnes. The Vlasov-fluid model assumes that the ions are described by a Vlasov equation, and that the electrons are treated as a cold, massless fluid. Since MHD and the Vlasov-fluid model have the same marginal point[3,4], we would not calculate complete kinetic stabilization if all terms were evaluated exactly. However, in an FLR theory, where particle resonances are neglected, it is possible to obtain complete stabilization[2].

In the Vlasov-fluid model the scalar and vector potentials are represented in terms of a displacement vector $\vec{\xi}$ as $\vec{A}_1 = \vec{\xi}\times\vec{B}$ and $\varphi_1 = \vec{\xi}\cdot\vec{E}$. The dispersion functional $\Delta(\vec{\xi}^*,\vec{\xi})$ is obtained by multiplying the equations of motion of the Vlasov-fluid model by $\vec{\xi}^*\cdot$ and integrating over all space. The result from Seyler and Barnes is

$$\Delta = -2\delta W - \omega F + 2\omega^2 K - R(\omega) = 0. \tag{1}$$

In the last equation $\delta W$ is the incompressible ideal MHD $\delta W$, F is the FLR term, K is the kinetic energy normalization term, and $R(\omega)$ is the resonant particle term. $R(\omega)$ is neglected in an FLR theory, where $\varepsilon=\rho_{i0}/a<<1$ and $\delta=a/b<<1$, where a and b are the minor and major radii, respectively, of the FRC, and $\rho_{i0}$ is the ion Larmor radius at the wall.

It can be shown that the terms K and F of Seyler and Barnes are incomplete in that $R(\omega)$ contains terms that are of the same order as those retained in K and F. The addition of the new terms leads to a new dispersion functional

$$\Delta_2 = -2\delta W - \omega F_2 + 2\omega^2 K_2 + O(\epsilon^4\delta,\epsilon^3\delta^2) = 0. \qquad (2)$$

From Eq. (74) of Seyler and Barnes, the term F is of the form

$$F = 2\pi \int_{\psi_0}^{0} d\psi \left(b_1 \left|\frac{d\xi_z}{d\psi}\right|^2 + b_2 |\xi_z|^2\right), \qquad (3)$$

where $b_2(\psi)$ is of the form

$$b_2(\psi) = P(\psi) \oint \frac{ds}{B\Omega_n}\, \beta_2(s,\psi). \qquad (4)$$

We have followed Seyler and Barnes by replacing the local gyrofrequency $\Omega$ by a nonlocal frequency, $\Omega_n$, which transfers smoothly from $\Omega$ to the betatron frequency as one approaches the field null. It is easy to show that the new FLR term merely leads to a modification of $\beta_2$ in Eq. (4):

$$\beta_2(s,\psi) \rightarrow \beta_2(s,\psi) - n\hat{B}_z^2\left[\frac{rB}{P}P'(\psi) + \frac{\nabla_\perp B}{B} - 3\kappa_\parallel\right]/r. \qquad (5)$$

As can be seen from (5) the new terms of $F_2$ arise from a particular flux surface average of the single particle drift frequency, and these new terms are important numerically compared to the terms in F.

When the dispersion functional $\Delta_2$ was used in the eigenvalue code of reference 1, it was discovered numerically and then understood analytically that the FLR terms were becoming infinite at the magnetic axis. Recall that Seyler and Barnes avoided one singularity by replacing $\Omega$ by the nonlocal frequency $\Omega_n$ in Eqs. (4-5). However, there still remain divergences in $F_2$. As $\psi \rightarrow \psi_0$ the flux surfaces become (highly elongated) ellipses, and it is possible to derive analytical forms for the curvatures $\kappa_\parallel$ and $\kappa_\perp$. It is then possible to show that terms such as the last term in the square brackets of (5) diverge as $\psi \rightarrow \psi_0$. The effect of this is that near the field null the FLR term completely dominates the unstable $\delta W$ term in the dispersion functional, and "FLR stabilization" of the tilting mode is achieved. Obviously this is not a physical result. One might argue that it would be more realistic to cut off the divergences in $F_2$ at some cut-off value of $\psi$ near $\psi_0$, $\psi_c$; this would still allow the FLR terms to operate where presumably they are accurate, in the region away from the field null. This was done and in Table 1 we show the FLR growth rate normalized to the MHD value for various values of $\psi_c/\psi_0$.

| $\psi_c/\psi_0$ | 0.0 | 0.5 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|
| $\tilde{\gamma}$ | 1.00 | 0.86 | 0.63 | 0.14 | 0.00 | 0.00 |

TABLE 1. $\tilde{\gamma}$ is the FLR growth rate normalized to the MHD value and $\psi_c/\psi_0$ is the ratio of the value of $\psi$ where the FLR terms are cutoff to the total trapped flux.

Table 1 shows that there __is__ a significant FLR stabilizing effect from the flux surfaces away from the field null (say, $\psi_c/\psi_0 < 0.8$). However we must question the reliability of the results in Table 1. There always are ways to smooth out singularities, but to do so is __ad hoc__ and physically may not be unique. Table 1 shows that the normalized growth rate can have any value between 0 and 1, depending on just where the FLR terms are cut off. Equation (2) can be solved for $\omega$: $\omega = [F_2 \pm (F_2^2 + 16\delta W K_2)^{1/2}]/4K_2$. From this equation we see that the FLR term $F_2$ is always stabilizing (for eigenfunctions $\xi$ not too different from the MHD eigenfunction). Since __any__ expression for $F_2$ is stabilizing, we should have confidence in the results only when $F_2$ is obtained rigorously and in a parameter regime where the FLR approximation is a good one. To summarize, if the full FLR terms are kept the result is unphysical, FRX-C. In Figs. 1c and 1d are the corresponding plots for 5 mTorr operation of FRX-C, in which the ion temperature is 600 eV ($s=2.4$). In the low temperature case the magnetic moment of the ion is conserved well in the straight field line region of the FRC, and in the high temperature case the oscillation in the magnetic moment is ±15%. In both cases when the ion reaches the tips of the flux surfaces the magnetic moment experiences large, random oscillations; this is caused primarily by the fact that the magnetic field line curvatures are large enough compared to the Larmor radius that the ions experience a nonadiabatic shift from $v_\perp$ to $v_\parallel$. Also, Fig. 1b shows that after the ion experiences the nonadiabatic region at the tips of the flux surfaces its magnetic moment jumps to quite a different value than it started with. This behavior is particularly unacceptable in the case of the tilting mode where the eigenfunction is peaked at the tips of the flux surfaces. The trajectories shown in Figs. 1a and 1c were not specially chosen—we have inspected many trajectories, even for subthermal particles, and $\mu$ is always poorly conserved as in Figs. 1b and 1d. Also not shown are those trajectories that pass near the field null, where the definition of $\mu$ becomes meaningless (this is also the region of peak ion number density).

Following a suggestion by R. Spencer, we can ask what is the critical value of s, $s_{crit}$, such that for $s>s_{crit}$ $\mu$ is conserved satisfactorily (say, $\Delta\mu<\pm25\%$) for a thermal ion away from the field null? $s_{crit}$ may give a rough indication of when FLR fluid effects become realizable. In Figs. 2a and 2b we show two elliptical equilibria[6], with $x_s=0.59$ and $x_s=0.87$, respectively, and in 2c and 2d two racetrack equilibria, with $x_s=0.60$ and $x_s=0.90$, respectively. The values of $s_{crit}$ are shown in the Figure. The conclusion is that FLR fluid effects may not become realizable until s exceeds 30 for equilibria of moderate $x_s$ ($x_s<0.6$) and 45–60 for equilibria of large $x_s$ ($x_s>0.85$).

## V. CONCLUSIONS

We have shown that an FLR theory of the tilting mode in FRCs is unreliable due to a breakdown of the FLR assumptions of small ion Larmor radius and constant magnetic moment. If the answer to the FRC tilting problem lies in linear stability theory, then evidently a nonlocal kinetic stability computation must be done, and we will report on that calculation in a future publication.

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if the FLR terms are cut off at some value of $\psi$ the resulting growth rate is not unique since it reflects that arbitrary cut-off.

This discouraging conclusion arises unavoidably from an FLR theory, and this is one reason why we feel FLR theories are not appropriate for global modes in FRCs. In an FLR theory it is assumed that the local Larmor radius of the particles is small compared to any other scale length of the problem, such as the scale length of the normal mode. Thus in the orbit integrals, which cover a spatial region roughly the size of the local ion Larmor (or betatron) radius, the perturbation $\xi_z$ is pulled outside the integral. However, the spatial extent of an ion orbit is not a small quantity near the field null of an FRC. The exact orbit integrals contain curvature factors that are averaged both along the field line, as in (4-6), and across various flux surfaces, as determined by the spatial extent of the local ion orbit. It is the latter averaging that yields a finite result for the exact orbit integral but which is absent in the FLR approximation.

## IV. CONSTANCY OF THE MAGNETIC MOMENT

In this section we present numerical results for the constancy of the magnetic moment for ions in FRC equilibria. A convenient parameter that measures the number of ion Larmor radii from the field null to the separatrix is[5]

$$s = \int_R^{r_s} \frac{dr\, r}{r_s \rho_i(r)} \approx \frac{x_s R}{5\rho_{i0}}, \tag{6}$$

where $x_s$ is the ratio of separatrix radius to wall radius ($x_s$=.59 for our equilibrium).

We solve the equations of motion numerically for an ion in the two dimensional effective potential of an FRC. As the particle trajectory is computed we monitor the instantaneous magnetic moment of the particle, $\mu(t) = mv_\perp^2/2B_{gc}$, where $B_{gc}$ is measured at the particle's instantaneous guiding center position and $v_\perp$ is measured in the particle's local $\hat{E} \times \hat{B}$ frame.

In Fig. 1a we show a plot of a thermal ion trajectory superimposed on the flux surfaces of the equilibrium being considered in this paper. In Fig. 1b is the corresponding plot of $\mu$ normalized to its value at t=0, as a function of time along the trajectory. These two plots correspond to an ion temperature of 100 eV (s=6.0), which is typical of 20 mTorr operation of

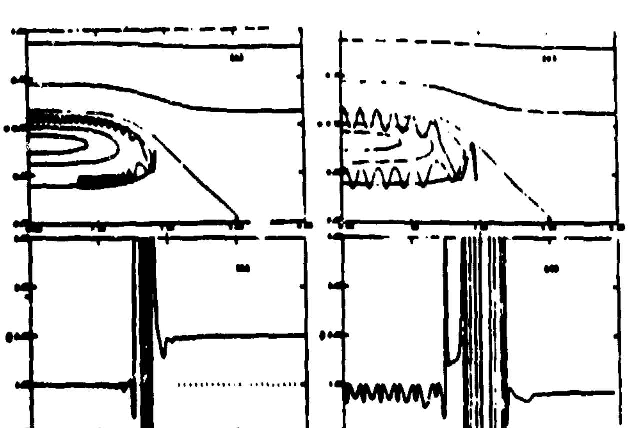

FIGURE 1. Particle trajectories and corresponding magnetic moments vs time.

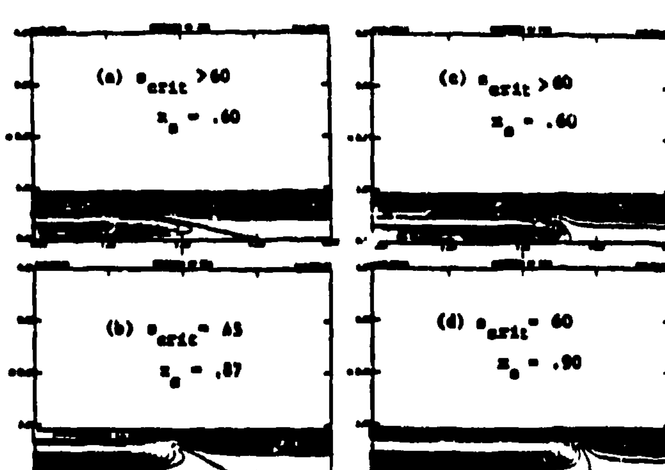


FIGURE 2. Elliptical and racetrack equilibria with critical s values.